PYTHAGORAS AND TRIGONOMETRY
Geometry and Measures

Key Concepts

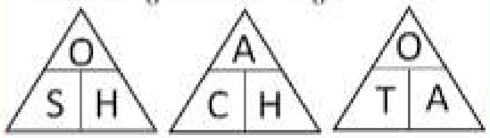
Pythagoras' theorem and basic trigonometry both work with **right angled triangles.**

Pythagoras' Theorem – used to find a missing length when two sides are known

$$a^2 + b^2 = c^2$$

c is always the hypotenuse (the longest side)

Basic trigonometry SOHCAHTOA – used to find a missing side or an angle

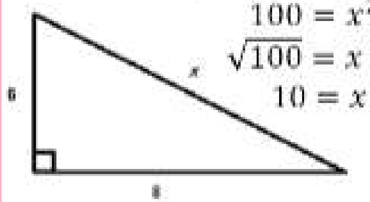


When finding the missing angle we must press **SHIFT** on our calculators first.

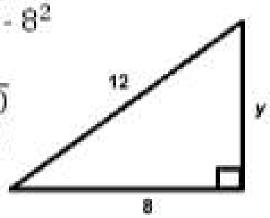
Examples

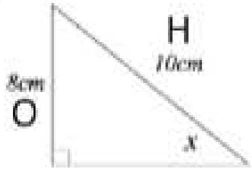
Pythagoras' Theorem

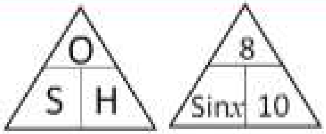
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$



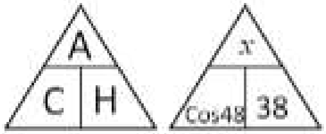
$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 12^2$$
$$a^2 = 12^2 - 8^2$$
$$a^2 = 80$$
$$a = \sqrt{80}$$
$$a = 8.9$$



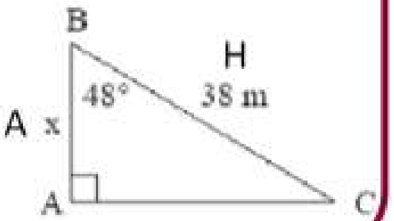




$$\sin x = \frac{8}{10}$$
$$x = \sin^{-1}\left(\frac{8}{10}\right)$$
$$x = 53.1^{\circ}$$



$$\cos 48 = \frac{x}{38}$$
$$38 \times \cos 48 = x$$
$$x = 25.4m$$



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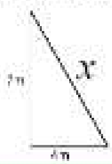
150 a,b,c, 168

Key Words

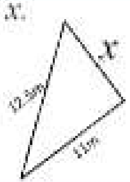
Right angled triangle
Hypotenuse
Opposite
Adjacent
Sine
Cosine
Tangent

Find the value of x.

a)



b)



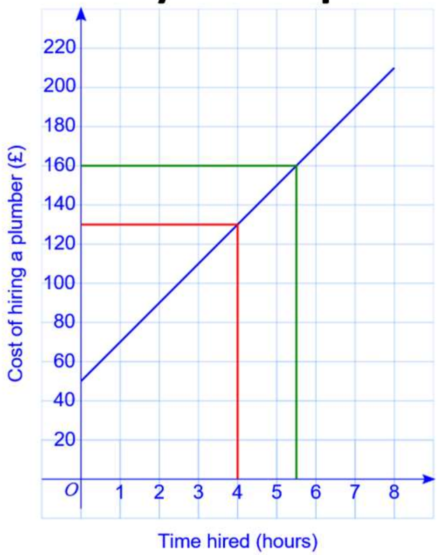
c)

d)

ANSWERS: a) 8.06m b) 5.94m c) 55.15° d) 2.34cm

APPLIED GRAPHS
Algebra

Key Concept



Gradient – The extra cost incurred for every extra hour.
y-intercept – The minimum payment to the plumber.

Key Words

Conversion graph: A graph which converts between two variables.
Intercept: Where two graphs cross.
y-intercept: Where a graph crosses the y-axis.
Gradient: The rate of change of one variable with respect to another. This can be seen by the steepness.
Simultaneous: At the same time.

Examples

What is the minimum taxi fair?
£2, this is the y-intercept.

What is the charge per mile?
50p, every extra mile adds on 50p.

How much would a journey of 5 miles cost?
£4.50, See line drawn up from 5 miles to the graph, then drawn across to find the cost.

Tip

The solution to two linear equations with two unknowns is the coordinates of the intercept (where they cross).

Questions

1) For the graph above a) A journey is 8 miles, what is its cost?
b) A journey cost just £3, how far was the journey?
2) Draw a graph to show the exchange rate £1 = $1.4.

ANSWERS: 1) a) £6 b) 2 miles

DISTANCE-TIME GRAPHS
Algebra

Key Concepts

A **distance-time** graph plots time against the distance away from a starting point.

Speed can be calculated from these graphs by finding the gradient of the graph.

Horizontal lines are sections where the object is stationary.

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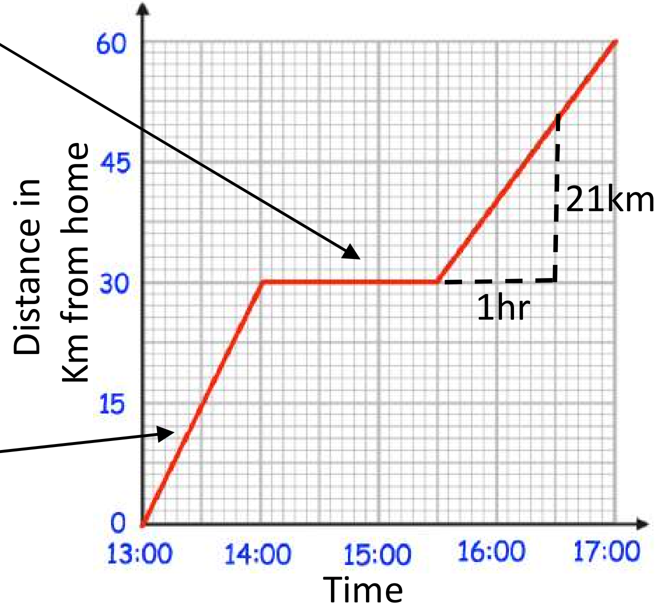
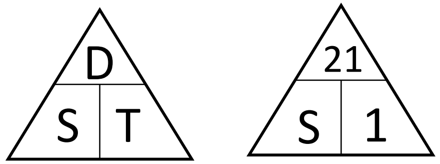
Examples

Horizontal sections are where the object is stationary

Diagonal lines show the object moving away from home or moving closer to home





$$Speed = \frac{distance}{time}$$

$$Speed = \frac{21}{1}$$

$$Speed = 21km/h$$

Key Words

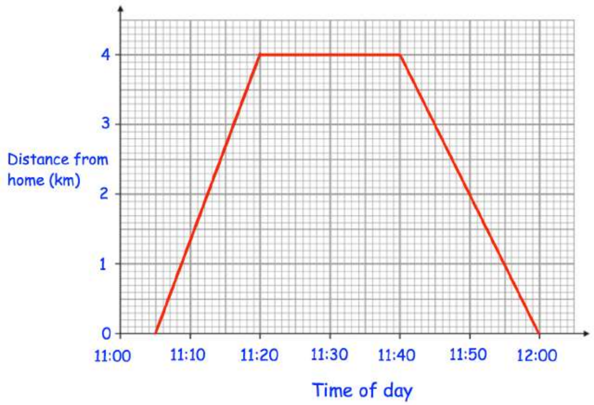
Distance

Time

Speed

Gradient

Stationary



A distance-time graph shows the journey of someone from home to the shop and back again.

1) How long were they at the shop for?
2) How far away from home is the shop?
3) How far did they travel in total?
4) What speed did they travel on the way to the shop in km/h?

ANSWERS: 1) 20 minutes 2) 4km 3) 8km 4) 16km/h

EQUATION OF A LINE BETWEEN TWO POINTS
Algebra

Key Concepts

Equation of a line is usually seen in the format:

$$y = mx + c$$

m = gradient
c = y-intercept

Examples

Find the equation of the line between the coordinates (1,1) and (3,5).

$$y = mx + c$$

$$m = \frac{5-1}{3-1} = \frac{4}{2} = 2$$

$$y = 2x + c$$

Substitute in one of the coordinates to find c

I have chosen to substitute in (3,5).

$$5 = (2 \times 3) + c$$

$$-1 = c$$

$$y = 2x - 1$$

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159b

Key Words
Gradient
Intercept
Equation

1) Find the equation of the line between the coordinates (2,5) and (5,11).
2) Find the equation of the line between the coordinates (5,3) and (7,11).

ANSWERS: 1) $y = 2x + 1$ 2) $y = 4x - 17$

EXPAND AND SIMPLIFY BRACKETS
Algebra

Key Concepts

Expanding brackets
Single: Where each term inside the bracket is multiplied by the term on the outside of the bracket.
Double: Where each term in the first bracket is multiplied by all terms in the second bracket.

Factorising expressions
Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Difference of two squares
When two brackets are repeated with the exception of a sign change. All numbers in the original expression will be square numbers.

Examples

Linear expressions

Expand and simplify where appropriate

1) 7 (3 + a) = 21 + 7a

2) 2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a
= 5a + 16

3) Factorise $9x + 18$ = $9(x + 2)$

4) Factorise $6e^2 – 3e$ = $3e(2e – 1)$

Quadratic expressions

Expand and simplify:

1) $(p + 2)(2p – 1)$
$= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

2) $(p + 2)^2$
$(p + 2)(p + 2)$
$= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Factorise:

3) $x^2 - 2x - 3$
$= (x - 3)(x + 1)$

Factorise and solve:

4) $x^2 + 4x - 5 = 0$
$(x - 1)(x + 5) = 0$
Therefore the solutions are:
Either $x - 1 = 0$
$x = 1$
Or $x + 5 = 0$
$x = -5$

MathsWatch

93, 94, 134a, 134b, 157, 158

Key Words
Expand
Factorise
Simplify
Product
Solve

1) Expand and simplify (a) 3(2 – 7f) (b) 5(m – 2) + 6 (c) 3(4 + t) + 2(5 + t)

2) Factorise (a) 6m + 12t (b) 9t – 3p (c) $4d^2 – 2d$

3) Expand (5g – 4)(2g + 1)

4) (a) **Factorise** $x^2 – 8x + 15$ **(b) Factorise and solve** $x^2 + 7x + 10 = 0$

ANSWERS: 1) (a) 6 – 21f (b) 5m – 4 (c) 22 + 5t 2) (a) 6(m + 2t) (b) 3(3t – p) (c) 2d(2d – 1)
3) $10g^2 – 3g – 4$ 4) (a) $(x – 3)(x – 5)$ (b) $x = -2$ or $x = -5$

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

Algebra

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

1) $5(y + 6) \equiv 5y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side
2) $5m - 7$ is an expression since there is no equals sign
3) $3x - 6 = 12$ is an equation as it can be solved to give a solution
4) $C = \frac{5(F - 32)}{9}$ is a formula (involves more than one letter and includes an equal sign)
5) Find the value of $3x + 2$ when $x = 5$

   $(3 \times 5) + 2 = 17$
6) Where $A = b^2 + c$, find A when $b = 2$ and $c = 3$

   $A = 2^2 + 3$

   $A = 4 + 3$

   $A = 7$

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Key Words

Substitute

Equation

Formula

Identity

Expression

Questions

1) **Identify the equation, expression, identity, formula from the list** (a) $v = u + at$ (b) $u^2 - 2as$ (c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$

2) Find the value of $5x - 7$ when $x = 3$

3) Where $A = d^2 + e$, find A when $d = 5$ and $e = 2$

ANSWERS: 1) (a) formula (b) expression (c) identity (d) equation 2) 8 3) A = 27

PLOTTING AND INTERPRETING GRAPHS
Algebra

Key Concept

Substitution – This is where you replace a number with a letter

If a = 5 and b = 2

a + b =	5 + 2 = 7
a – b =	5 – 2 = 3
3a =	3 × 5 = 15
ab =	5 × 2 = 10
a^2 =	5^2 = 25

MathsWatch

95, 96, 97, 159a

Key Words

Intercept: Where two graphs cross.
Gradient: This describes the steepness of the line.
y-intercept: Where the graph crosses the y-axis.
Linear: A linear graph is a straight line.
Quadratic: A quadratic graph is curved, u or n shape.

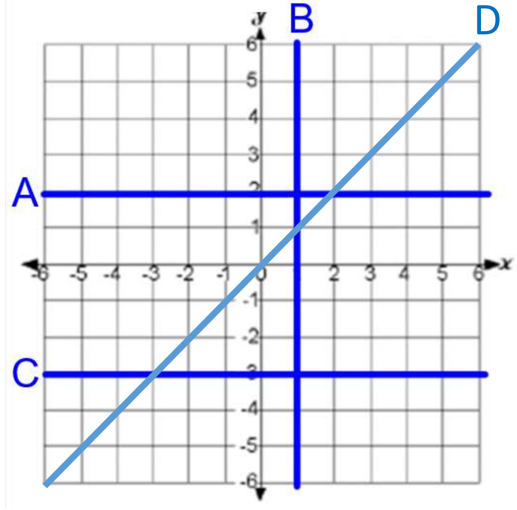
Tip

Parallel lines have the same gradient.

Formula

$$Gradient = \frac{difference\ in\ y's}{difference\ in\ x's}$$

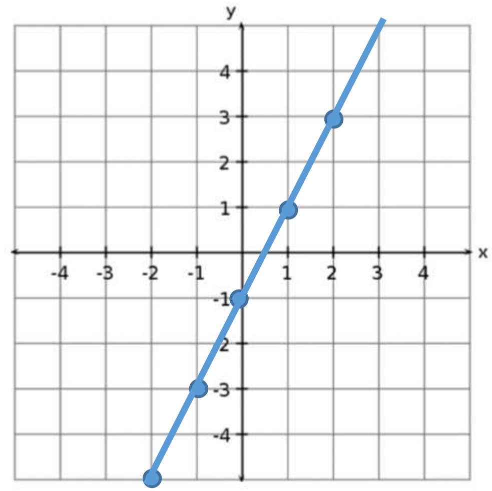
Examples



A: y = 2   B: x = 1
C: y = -3   D: y = x

Draw the graph of y = 2x - 1

X	- 2	- 1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.

Questions

1) What are the gradient and y-intercept of:
a) y = 4x – 3   b) y = 4 + 6x   c) y = - 5x – 3
2) Draw the graph of y = 3x – 2 for x values from -3 to 3 using a table.

ANSWERS: 1) a) m = 4, c = -3   b) m = 6, c = 4   c) m = -5, c = -3

REARRANGE AND SOLVE EQUATIONS
Algebra

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

For each step in solving an equation we must do the **inverse** operation

Solve:

$5(x - 3) = 20$

Expand

$5x - 15 = 20$

+15 +15

$5x = 35$

÷5 ÷5

$x = 7$

Solve:

$12 = 3x - 18$

+18 +18

$30 = 3x$

÷3 ÷3

$x = 10$

Solve:

$7p - 5 = 3p + 3$

-3p -3p

$4p - 5 = 3$

+5 +5

$4p = 8$

÷ 2 ÷ 2

$p = 2$

Examples

Rearrange to make r the subject of the formulae :

$$Q = \frac{2r - 7}{3}$$

×3 × 3

$$3Q = 2r - 7$$

+7 +7

$$3Q + 7 = 2r$$

÷ 2 ÷ 2

$$\frac{3Q + 7}{2} = r$$

MathsWatch

100, 135a, 135b

Key Words
Solve
Rearrange
Term
Inverse
operation

1) Solve $7(x + 2) = 35$
2) Solve $4x - 12 = 28$
3) Solve $4x - 12 = 2x + 20$
4) Rearrange to make x the subject:

$$y = \frac{3x + 4}{2}$$

ANSWERS: 1) $x = 3$ 2) $x = 10$ 3) $x = 16$ 4) $x = \frac{2y - 4}{3}$

REARRANGING EQUATIONS
Algebra

Key Concepts

Rearranging an equation: Working with inverse operations to isolate a highlighted variable.

When rearranging we **undo the operations** starting from the last one.

Examples

Rearrange to make r the subject of the formulae :

$$Q = \frac{2r-7}{3}$$

×3   × 3

$$3Q = 2r - 7$$

+7   +7

$$3Q + 7 = 2r$$

÷ 2   ÷ 2

$$\frac{3Q+7}{2} = r$$

Rearrange to make c the subject of the formulae :

$$2(3a - c) = 5c + 1$$

expand

$$6a - 2c = 5c + 1$$

$+2c$ $+2c$

$$6a = 7c + 1$$

-1   -1

$$6a - 1 = 7c$$

÷ 7   ÷ 7

$$\frac{6a-1}{7} = c$$

Rearrange to make a the subject of the formulae :

$$\sqrt{\frac{ac}{b}} = d$$

square   square

$$\frac{ac}{b} = d^2$$

$\times b$ $\times b$

$$ac = bd^2$$

$\div c$ $\div c$

$$a = \frac{bd^2}{c}$$

MathsWatch

136, 190

Key Words

Rearrange

Term

Inverse

1) Rearrange to make a the subject $r = \frac{5a+3}{t}$

2) Rearrange to make m the subject $2(2p + m) = 3 - 5m$

3) Rearrange to make x the subject $\sqrt{\frac{4x}{y}} = z$

ANSWERS: 1) $a = \frac{rt-3}{5}$ 2) $m = \frac{3-4p}{7}$ 3) $x = \frac{yz^2}{4}$

SEQUENCES
Algebra

Key Concepts

Arithmetic or linear sequences increase or decrease by a common amount each time.

Geometric series has a common multiple between each term.

Quadratic sequences include an n^2. It has a common second difference.

Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequence:

-3   +3   +3   +3   +3

+1   4   7   10   13   16.....

a) State the nth term

$3n + 1$

Difference   The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

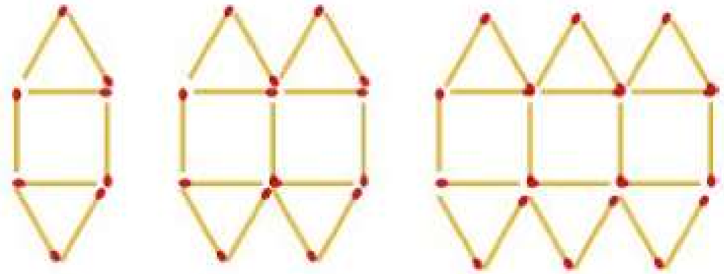
$3n = 99$

$n = 33$

Yes as 33 is an integer.

Examples

Pattern 1   Pattern 2   Pattern 3



Linear sequences with a picture:

State the nth term.

Hint: Firstly write down the number of matchsticks in each image:

$7n + 1$

Pattern 1	Pattern 2	Pattern 3
8	15	22

+1   -7   +7   +7

Geometric sequence e.g.   ×3   ×3   ×3

4   12   36   108...

Quadratic sequence e.g. $n^2 + 4$ Find the first 3 numbers in the sequence

First term: $1^2 + 4 = 5$

Second term: $2^2 + 4 = 8$

Third term: $3^2 + 4 = 13$

MathsWatch

102, 103, 104

Key Words

Linear

Arithmetic

Geometric

Sequence

Nth term

1) 1, 8, 15, 22, ...

a) Find the nth term   b) Calculate the 50th term   c) Is 120 in the sequence?

2) $n^2 - 5$ Find the first 4 terms in this sequence

ANSWERS: 1a) $7n - 6$ b) 344   c) 18 so yes as n is an integer   2) -4, -1, 4, 11

SIMPLIFYING & MANIPULATING ALGEBRA

Algebra

Key Concept

Formula

$$v = u + at$$

Expression

$$f^2 + f^2 + f^2$$

Equation

$$34 = 12 + 6t$$

Identity

$$c \times c = c^2$$

MathsWatch

33, 93, 94, 134a

Key Words

Formula: A rule written using symbols that describe a relationship between different quantities.
Expression: Shows a mathematical relationship whereby there is no solution.
Equation: A mathematical statement that shows that two expressions are equal.
Identity: A relation which is true. No matter what values are chosen.

Tip

When expanding brackets be careful with negatives.

Examples

Simplify:

4a + 3b − a + 2b

= 3a + 5b

Expand and simplify:

$$9a - 2(3a - 4)$$

$$9a - 6a + 8$$

$$3a + 8$$

Factorise:

$$9x^2 + 6x$$

Factorising is the opposite of expanding brackets

$3x$ is common to both terms

$$3x(3x + 2)$$

Expand and simplify:

$$2(4a + 2b) - 2(a + 3b)$$

$$8a + 4b - 2a - 6b$$

$$6a - 2b$$

Questions

1) $5x + 3y - 2x + 4y$ 2) $2p - 6q + 2q + 4p$ 3) $12b - 3(2b + 5)$
4) Factorise a) $4x + 10$ b) $8a^2 - 10a$

ANSWERS: 1) $3x + 7y$ 2) $6p - 4q$ 3) $6b - 15$
4) a) $2(2x + 5)$ b) $2a(4a - 5)$

STRAIGHT LINE GRAPHS AND EQUATION OF A LINE

Algebra

Key Concepts

Coordinates in 2D are written as follows:

(x, y)

x is the value that is to the left/right

y is the value that is to up/down

Straight line graphs always have the equation:

$$y = mx + c$$

m is the **gradient** i.e. the steepness of the graph.

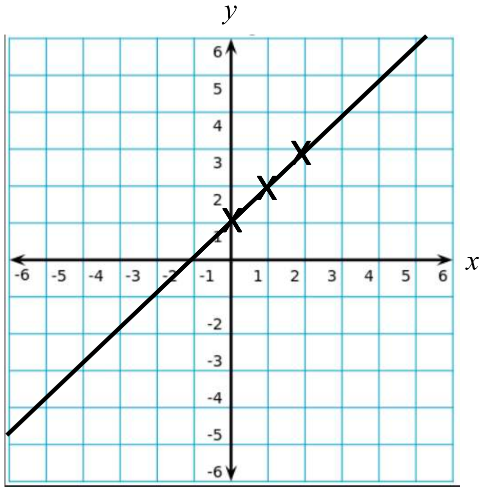
c is the **y intercept** i.e. where the graph cuts the y axis.

Parallel lines always have the same **gradient**.

Examples

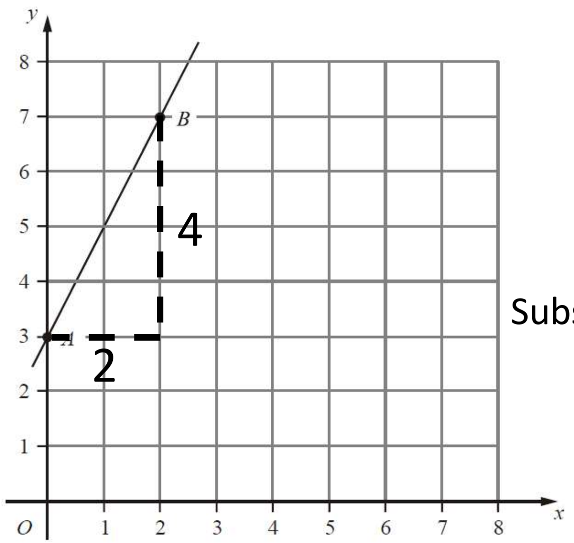
Plot the graph of $y = 2x + 1$

x	**0**	**1**	**2**
y	1	2	3



Examples of lines parallel to this graph are: $y = 2x - 3$ or $y = 2x + 7$

Calculate the equation of this line:



$$y = mx + c$$

$$m = \frac{4}{2} = 2$$

$$y = 2x + c$$

Substitute in a coordinate: (2,7)

$$7 = (2 \times 2) + c$$

$$3 = c$$

$$y = 2x + 3$$

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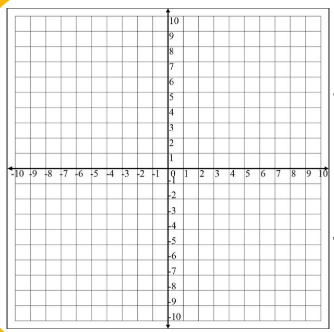
95, 96, 97, 159a

Key Words

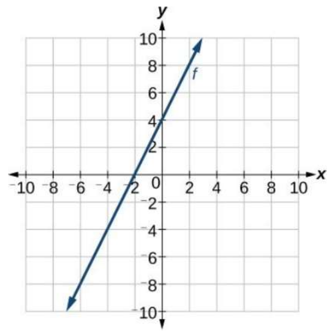
Coordinate

Gradient

Parallel



1) Plot the line $y = 3x - 2$
2) Find the equation of the line for the attached graph.
3) State the equation of a line that would be parallel to this line.



ANSWERS: 2) $y = 2x + 4$ 3) Any line that has a gradient of 2